Various processes

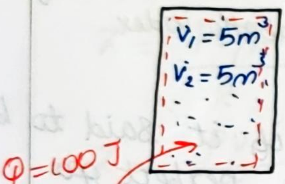
1) Constant Volume process (Stoppers)
2) Constant pressure process (Dead weight const)
3) Constant Temperature process (Remoing Rice grain and supplying heat)
4) Adiabatic process Removing Rice grain No supply of heat.
5) polytropic process.

1) CONSTANT VOLUME PROCESS

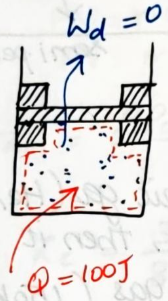
V = constant (Any gas) → Ideal gas
→ Real gas

Isochoric process.
Isometric process.

Physical representation



Isochoric heat addition



Due to heat stored

Energy ↑

Temperature ↑

$K.E = \frac{3}{2} KT$ ↳ Boltzman constant

$\frac{1}{2} mc^2 = \frac{3}{2} KT$

C ↑ T ↑

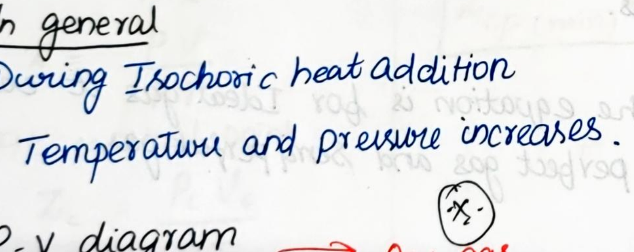
↳ Velocity ↑ due to heat supply.

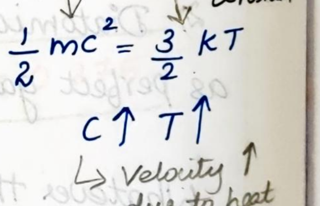
In general

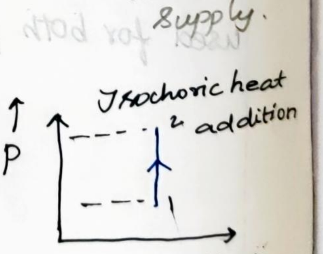
During Isochoric heat addition

Temperature and pressure increases.

P-V diagram → Any gas

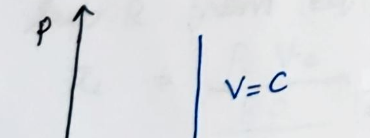


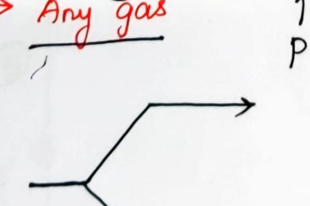




Isochoric heat rejection.

V-T diagram





Analysis for an ideal gas

According to ideal gas equation

$PV = mRT$

$P = \frac{mRT}{V}$

$\frac{P}{T} = \frac{mR}{V}$ ← const

$\frac{P}{T} = $ const

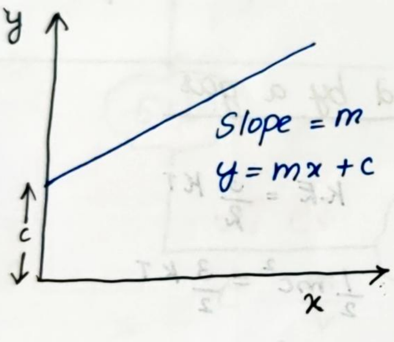
$\frac{P}{T} = $ constant — given mass, Ideal gas, Isochoric process

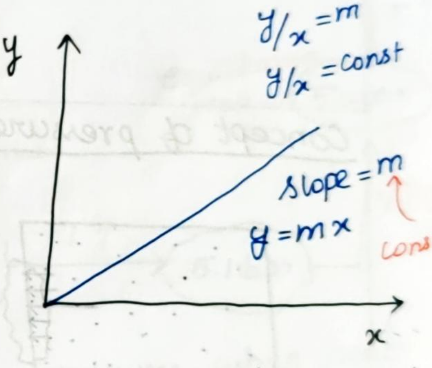
$P = \text{const} \times T$ $\quad P \propto T \quad$ $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Gay Lussac's law

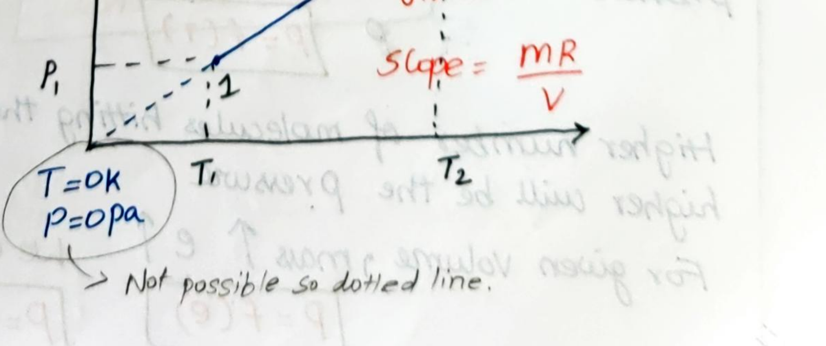
According to Gay Lussac's law, for a given mass of an ideal gas undergoing isochoric process, absolute pressure is directly proportional to absolute temperature.

Basic maths



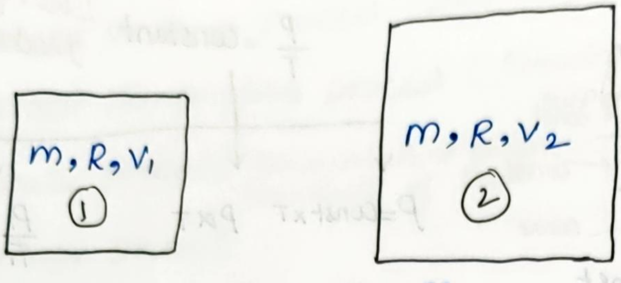


P-T diagram of ideal gas undergoing isochoric process

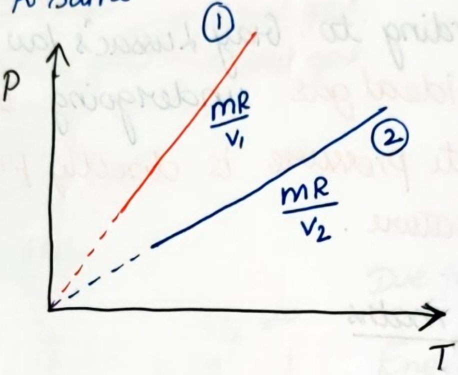


INTERVIEW QUESTION - 4

Draw the slope of PT diagram?



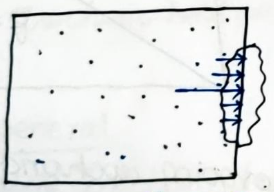
For given mass $m_1 = m_2 = m$
Same Ideal gas R Same
$V_1 < V_2$



Slope : $\frac{mR}{V}$

V↓ Slope↑

Concept of pressure exerted by a gas



Force exerted by molecules

$$K.E = \frac{3}{2} KT$$

$$\frac{1}{2} mc^2 = \frac{3}{2} KT$$

$$c^2 \propto T$$

Velocity of molecule

Absolute temperature.

If temperature increases, Velocity of molecules with which it strikes increases (ie) Force also increases. pressure also increases.

$$\boxed{P = f(T)}$$

Higher number of molecules hitting the wall, higher will be the pressure

$\rho = \frac{m}{V}$

For given volume, mass ↑ ρ ↑

$$\boxed{P = f(\rho)}$$

We can conclude that

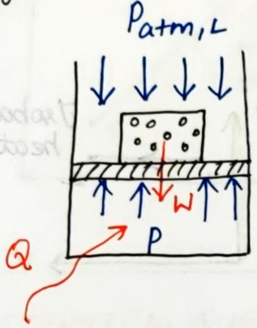
$$\boxed{P = f(T, \rho)}$$

2) CONSTANT PRESSURE PROCESS

$P = \text{constant}$ (Any gas)

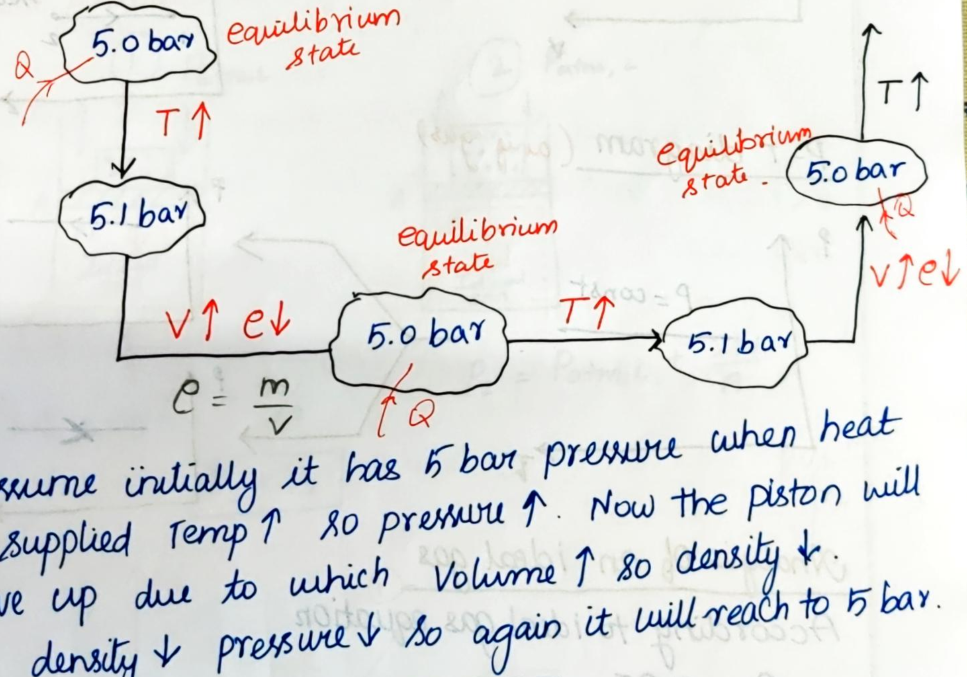
$$\boxed{\begin{array}{l}\text{Isobaric process.}\\ \text{Isopiestic process.}\end{array}}$$

Physical representation



By equilibrium

$$\boxed{PA = P_{atm,L}\,A + W}$$



Assume initially it has 5 bar pressure when heat is supplied Temp ↑ so pressure ↑. Now the piston will move up due to which Volume ↑ so density ↓. If density ↓ pressure ↓ so again it will reach to 5 bar.

$$\boxed{\begin{array}{l}\text{Temperature increases}\\ \text{Volume increases.}\end{array}}$$

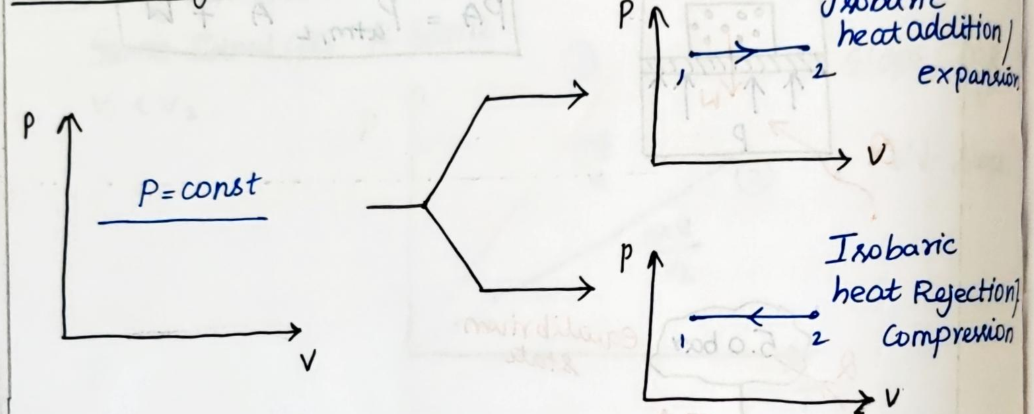
To prove Mathematically

$$PA = P_{atm,L}A + W \xrightarrow{} $$ This can be used only in equilibrium state because in other state pressure changes.

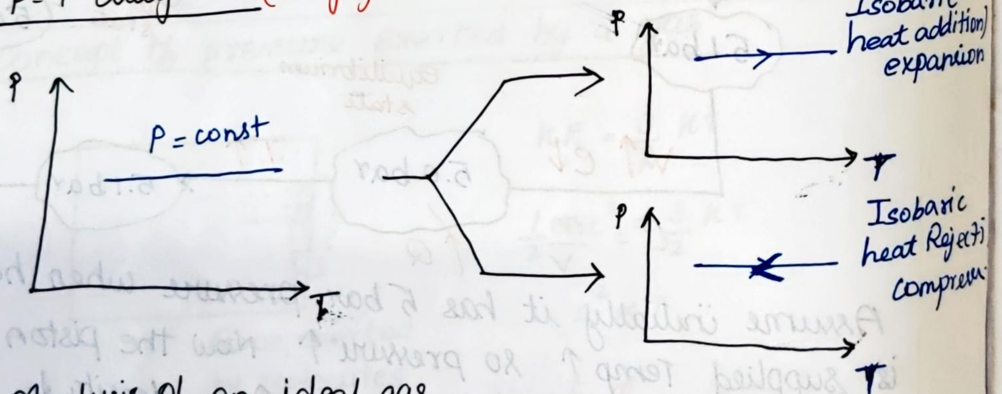
$$P = \underset{\uparrow\,const}{P_{atm,L}} + \overset{const}{\frac{W}{A}} \leftarrow const$$

$$\boxed{P = constant}$$

P-V diagram (any gas)



P-T diagram (any gas)



Analysis of an ideal gas

According to ideal gas equation

$$PV = mRT$$

$$\boxed{\frac{V}{T} = \frac{P}{mR}} \qquad V \propto T$$

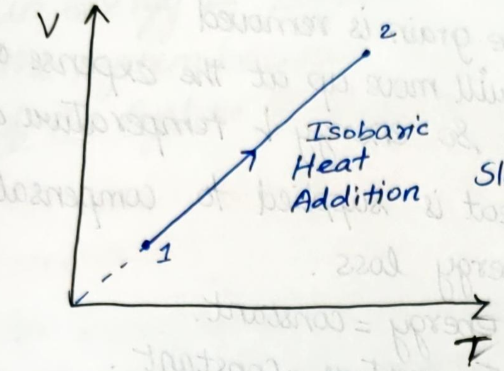
$$V/T = const \rightarrow$$ m = const, ideal gas, Isobaric process.

$$V = const \times T \qquad V \propto T \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\boxed{\text{Charles Law}}$$

According to Charles law for a given mass of an ideal gas undergoing isobaric process, Volume is directly proportional to absolute temperature.

V-T diagram for ideal gas undergoing Isobaric process



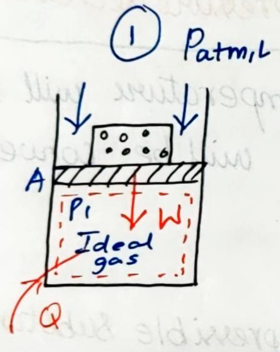
$V/T = const$

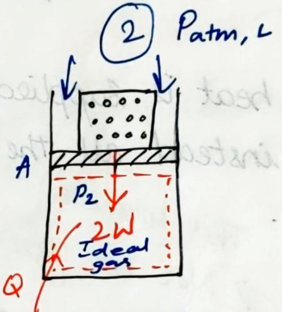
$PV = mRT$

$Slope = \frac{mR}{P}$

INTERVIEW QUESTION - 5

Which pressure is higher?





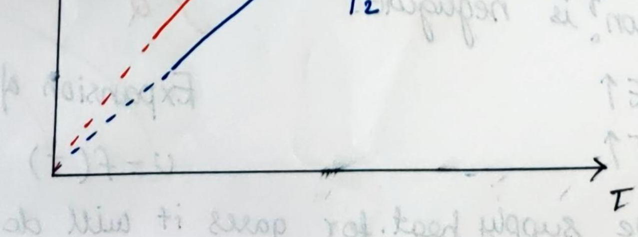
$P_1 = P_{atm,L} + W/A$

$P_2 = P_{atm,L} + \frac{2W}{A}$

$\boxed{P_1 < P_2}$

$PV = mRT$

$\boxed{V/T = C}$

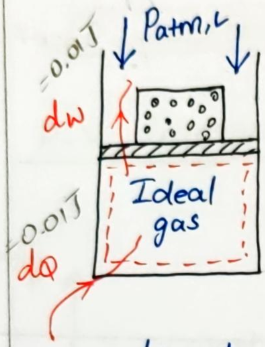


3) CONSTANT TEMPERATURE PROCESS

T = constant (any gas)

Isothermal process. → generally it is internally Reversible

Physical Representation



One rice grain is removed piston will move up at the expense of energy so energy ↓ temperature also ↓

Now heat is supplied to compensate the energy loss.

Now energy = constant.

Temperature = constant.

dQ = dw

$PV = mRT$

$P = \frac{mRT}{V}$

During Isothermal heat addition/Expansion

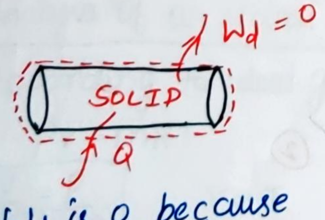
Volume increases
pressure decreases

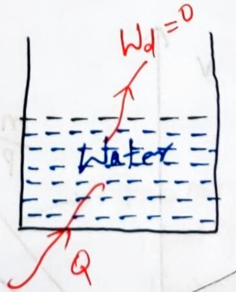
Though the heat is supplied temperature will not increase instead all the heat will be converted into work.

Note:-

$Q = mC\Delta T$ (only for Incompressible substance) → Not for gases
This formula

Solid   liquid





Wd is 0 because expansion of solid is negligible.

Here E↑

T↑

E↑

T↑

Expansion of water is negligible

U = f(T)

If we supply heat for gases it will do some work by expanding.

Conclusion

* on removal of rice grains, work is done by the ideal gas at the expense of energy. If at the same time, same amount of heat as that of decrease in energy is supplied then the final energy of the system becomes equal to the initial energy of the system leading to same initial temperature.

$U=f(T)$

* For ideal gas, heat can be supplied at constant temperature.

* For Incompressible substance (solid / liquids) on supplying heat, the energy of the system increases as workdone is negligible. Increase in energy of the system leads to increase in temperature.

* For an ideal gas undergoing isothermal process heat completely convert into work. → cannot be done for cyclic process

* Isothermal heat transfer to a closed system, neglection friction can be considered as an internally reversible process.

Note:

For Isochoric heat addition $V=const$; $P\uparrow, T\uparrow$

For Isobaric heat addition $P=const$; $V\uparrow, T\uparrow$

For Isothermal heat addition $T=const$; $V\uparrow, P\downarrow$

For Ideal gas $U=f(T)$ only if T is const U is also const

Analysis for an Ideal gas

According to ideal gas equation

$PV = mRT$

$PV = C$

$P \alpha \frac{1}{V}$

$$P \alpha 1/V$$

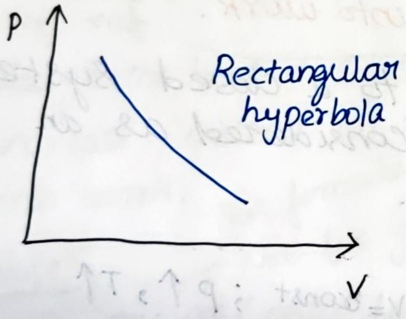
$PV = \text{const}$ $\quad$ $P_1 V_1 = P_2 V_2$

Boyle's law

According to Boyle's law for a given mass of an ideal gas undergoing isothermal process, absolute pressure is inversely proportional to Volume.

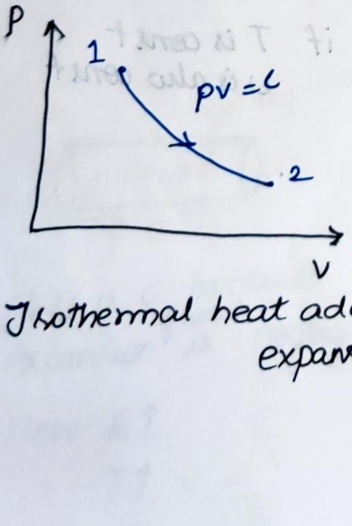
P-V diagram

only for ideal gas

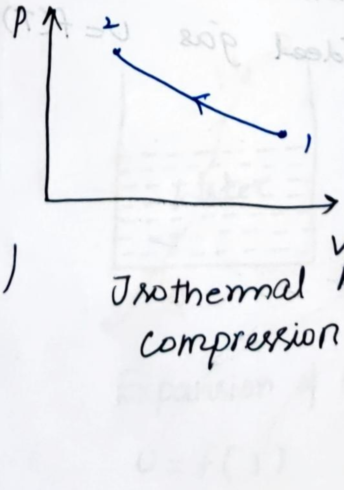


conclusion

For an ideal gas, Isothermal process on PV diagram is represented by a rectangular hyperbola which is symmetry to both pressure and Volume axis.



Isothermal heat addition / expansion



Isothermal heat rejection / compression.

Analysis

for an ideal gas undergoing Isothermal process.

$$pv = c$$

$$d(pv) = d(c)$$

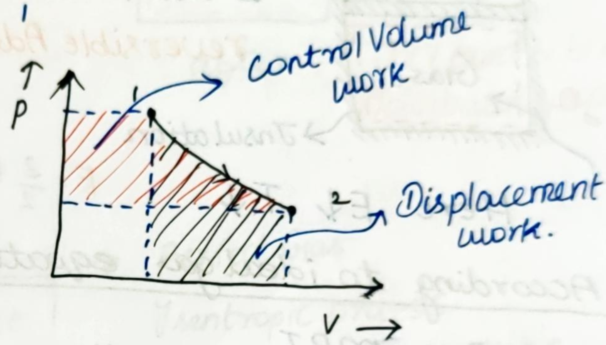
$$p\,dv + v\,dp = 0$$

$$p\,dv = -v\,dp$$

$$\int_1^2 p\,dv = -\int_1^2 v\,dp$$

Displacement work of closed system

Control Volume work of open system



Conclusion

For an ideal gas undergoing isothermal process. displacement work of closed System is equal to the control Volume work of open system provided initial and final states are same.

4) Adiabatic process

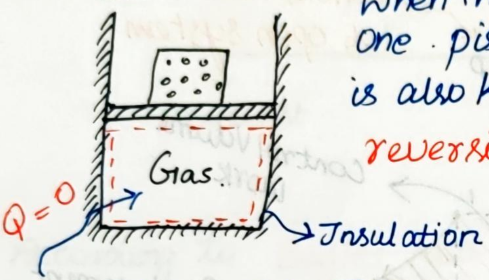
very fast process. If we keep hand in fire for negligible time it won't create any problem for hand

No heat interaction

$\boxed{Q=0}$ [Any gas]

Physical Representation

Adiabatic expansion



When the Rice grain is removed one by one. piston slowly moves up. So this is also known as internally reversible Adiabatic.

Here E↓ T↓ $E = \frac{3}{2}KT$

According to ideal gas equation

$$PV = mRT$$

$$P = \frac{mRT \rightarrow \text{decrease}}{V \rightarrow \text{Increase}}$$

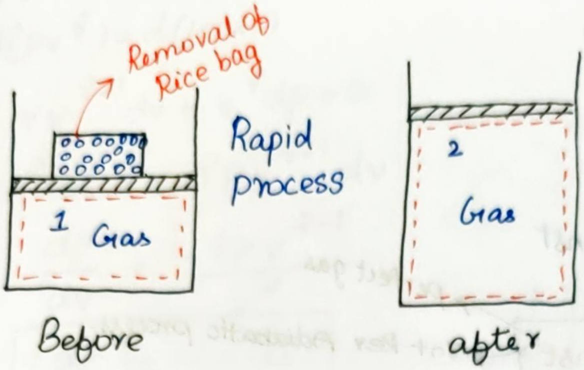
So P decreases

Difference between Isothermal / Adiabatic expansion.

Isothermal expansion	Adiabatic expansion
$P = \frac{mRT \leftarrow \text{const}}{V \leftarrow \text{Increase}}$	$P = \frac{mRT \leftarrow \text{decrease}}{V \leftarrow \text{Increase}}$
For the same increase in Volume	
Less decrease in pressure	More decrease in pressure.

Can Adiabatic process be done without insulating?

Yes it can be done.



$\dot{Q} = Q/t$

$Q = \dot{Q} \times t$

$t \to 0$

$Q = \dot{Q} \times \to 0$

$Q \to 0$

Irreversible Adiabatic expansion.

Various Relations

$\boxed{PV^{\gamma} = const}$ → perfect Gas
Isentropic process
(ie) internally reversible Adiabatic process.

To make γ = const
C_p and C_v should be const
So it has to be perfect gas

$\boxed{\gamma = \text{Adiabatic index}}$

$\gamma = \frac{C_p}{C_v} > 1$ Always greater than 1.

Interms of T and P

$PV^{\gamma} = const$

For perfect gas

$PV = mRT$

$V = \frac{mRT}{P}$

$P\left(\frac{mRT}{P}\right)^{\gamma} = const$

$P\frac{m^{\gamma}R^{\gamma}T^{\gamma}}{P^{\gamma}} = const$

$P^{1-\gamma}T^{\gamma} = const$

÷ power by γ $\quad TP^{\frac{1-\gamma}{\gamma}} = const$

$\boxed{TP^{\frac{1-\gamma}{\gamma}} = const}$ → perfect gas
→ Internally Rev Adiabatic process.

Interms of T and V

$$pv^{\gamma} = const$$

For perfect gas

$$pv = mRT$$

$$p = \frac{mRT}{v}$$

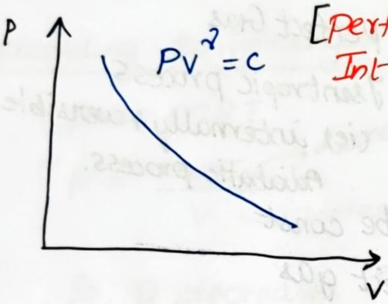
$$\left(\frac{mRT}{v}\right)v^{\gamma} = const$$

$$\boxed{Tv^{\gamma-1} = const}$$

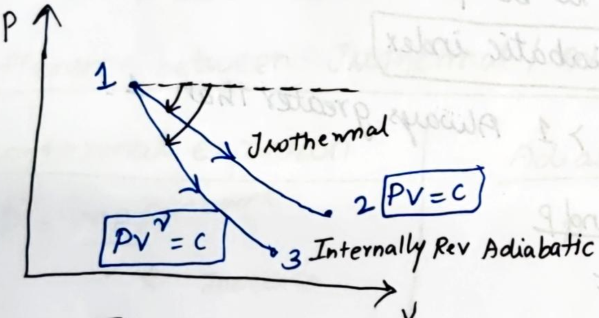
→ perfect gas

→ Int Rev Adiabatic process.

P-v diagram



[Perfect gas
Int Rev Adiabatic process]



Expansion

For Ideal Gas undergoing Isothermal process

$$pv = const$$

$$d(pv) = d(const)$$

$$pdv + vdp = 0$$

$$vdp = -pdv$$

$$\boxed{\left(\frac{dp}{dv}\right)_{isothermal} = -\frac{p}{v}}$$

For perfect gas undergoing internally Reversible Adiabatic proc

$pv^\gamma = const$

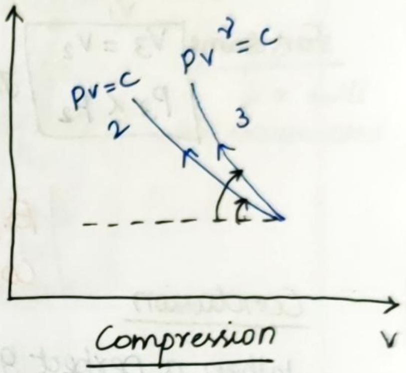
$d(pv^\gamma) = d(const)$

$p\gamma v^{\gamma-1} dv + v^\gamma dp = 0$

$v^\gamma dp = -\gamma p v^{\gamma-1} dv$

$$\frac{dp}{dv} = \frac{-\gamma p v^{\gamma-1}}{v^\gamma}$$

$$\boxed{\left(\frac{dp}{dv}\right)_{adiabatic} = -\gamma\frac{p}{v}}$$



Compression

Result $\left(\frac{dp}{dv}\right)_{adi} > \left(\frac{dp}{dv}\right)_{isothemal}.$

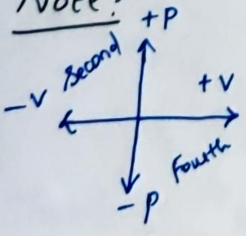
- sign because $p\uparrow, v\downarrow$
(or)
$p\downarrow, v\uparrow$

Conclusions

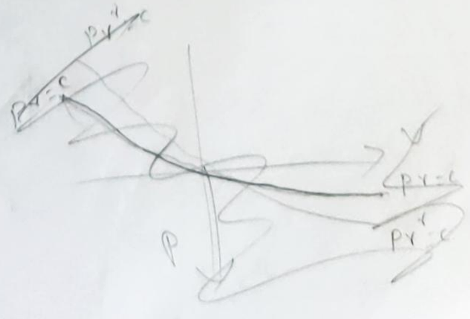
* For perfect gas on pv diagram, both isothermal and internally reversible adiabatic process have negative slopes.

* For perfect gas on same p-v diagram slope of internally reversible adiabatic process is more (γ times) than the slope of isothermal process.

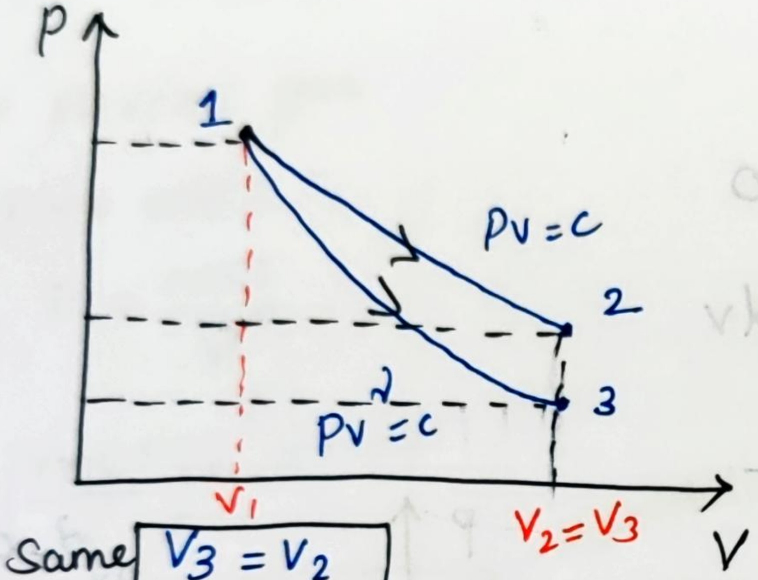
Note:



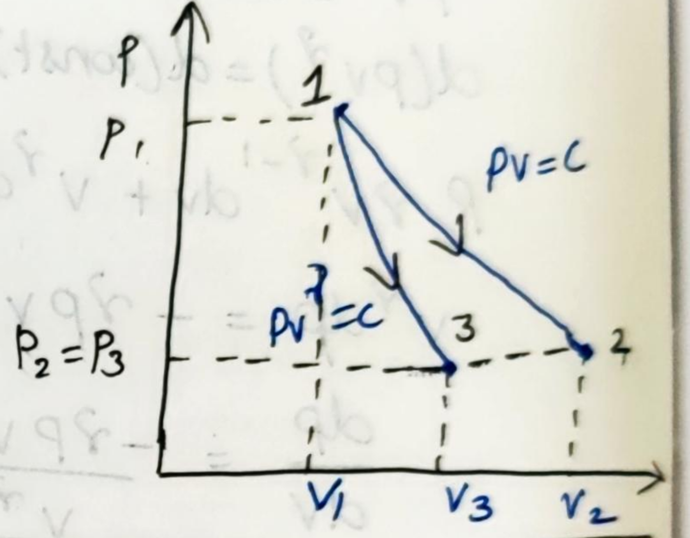
It will come in 2nd and 4th quadrant.



ANALYSIS



For Same $\boxed{\begin{array}{c} V_3 = V_2 \\ P_3 < P_2 \end{array}}$ Isothermal dominates



For same $\boxed{\begin{array}{c} P_2 = P_3 \\ V_2 > V_3 \end{array}}$ Isothermal dominates

Domination

Expansion → Isothermal
Compression → Rev Adiabatic

Conclusion

When a perfect gas is expanded isothermally and internally reversible adiabatically separately from same initial state

expansion

a) To same final volume $P_{iso} > P_{adiabatic}$

b) To same final pressure $V_{iso} > V_{adiabatic}$

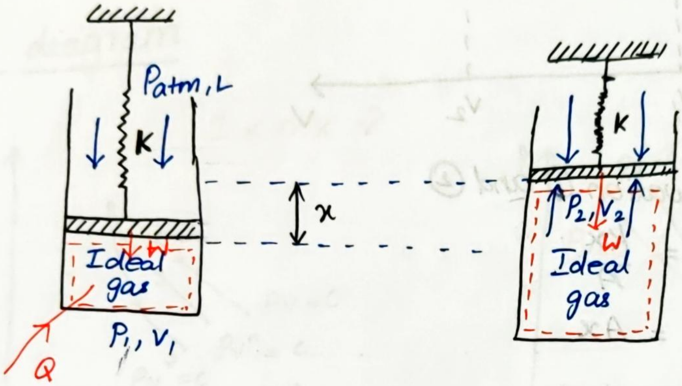
5) Polytropic process

Poly → Many
Tropic → Changes

$$\boxed{\begin{array}{c}\Delta V \neq 0 \\ \Delta P \neq 0 \\ \Delta T \neq 0 \\ Q \neq 0\end{array}}$$

No property will remain constant.

Physical Representation



Before

After

Spring force kx will act downward

$P_1 = P_{atm,L} + W/A$ — ①

$P_2 = P_{atm,L} + W/A + kx/A$ — ②

From ① and ②

$P_2 > P_1$ [Not Isobaric]

$PV = mRT$

$T = \frac{PV}{mR}$

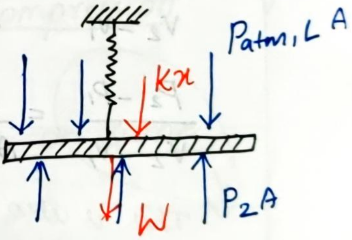
$T_1 = \frac{P_1 V_1}{mR}$ — ③

$T_2 = \frac{P_2 V_2}{mR}$ — ④

$P_2 , V_2 > P_1 V_1$

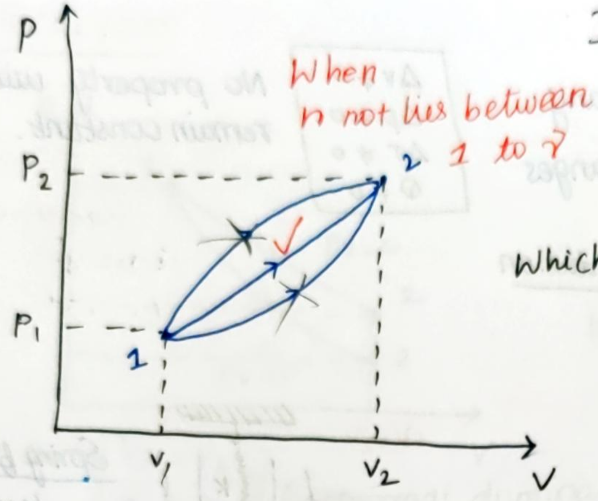
Therefore $T_2 > T_1$

[Not Isothermal]



$V_2 = V_1 + Ax$

$V_2 > V_1$ [Not Isochoric]



In polytropic process
heat addition
V↑ P↑ T↑

Which curve is correct?

From equation ① and ②

$$P_2 - P_1 = \frac{kx}{A}$$

$$V_2 - V_1 = Ax$$

$$\frac{P_2 - P_1}{V_2 - V_1} = \frac{kx/A}{A\cancel{x}}$$

$$\frac{P_2 - P_1}{V_2 - V_1} = \frac{k}{A^2}$$

→ const

→ const

This is also const

$$\frac{dp}{dv} = \text{const.}$$

(Slope)

Conclusion

- * An ideal gas enclosed by piston cylinder arrangement with spring undergoes polytropic process.
- * For the above polytropic process heat addition leads to increase in volume, pressure and temperature.

INTEVIEW QUESTION - 6

What will be the slope of pv diagram for polytropic heat addition to an ideal gas enclosed by piston cylinder arrangement with spring?

$$\text{Slope} = K/A^2$$

Equations for polytropic process

$pv^n = \text{const}$ [Any gas]

n is polytropic index

$-\infty \le n \le \infty$

In general $1 < n < \gamma$

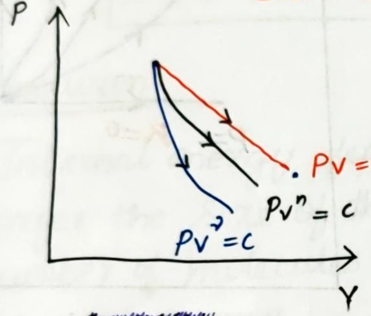
$Tv^{n-1} = \text{const}$

$Tp^{\frac{1-n}{n}} = \text{const}$ [Ideal gas]

↓ Because it comes from $pv = mRT$

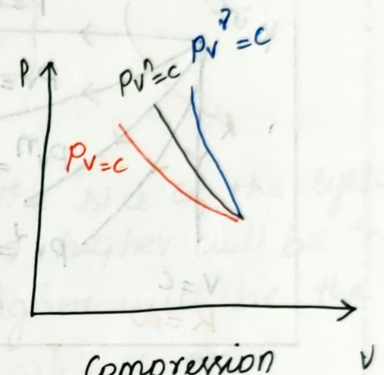
P-v diagram

$1 < n < \gamma$



Expansion

$1 < n < \gamma$



Compression

Representation of various processes on same pv diagram

$pv^k = \text{const}$.

1) Isochoric process

$pv^k = \text{const}$

$v = \text{const}$

$(pv^k)^{1/k} = (\text{const})^{1/k}$

$p^{1/k} v = \text{const}$

$p^{1/\infty \to 0} v = \text{const}$

$v = \text{const}$

$\boxed{k = \infty}$

2) Isobaric process

$pv^k = \text{const}$

$p = \text{const}$

$\boxed{k = 0}$

3) Isothermal process

$pv = c$

$pv^k = c$

$\boxed{k = 1}$

4) Adiabatic process

$q = 0$

$PV^{\gamma} = C$

$PV^{k} = C$

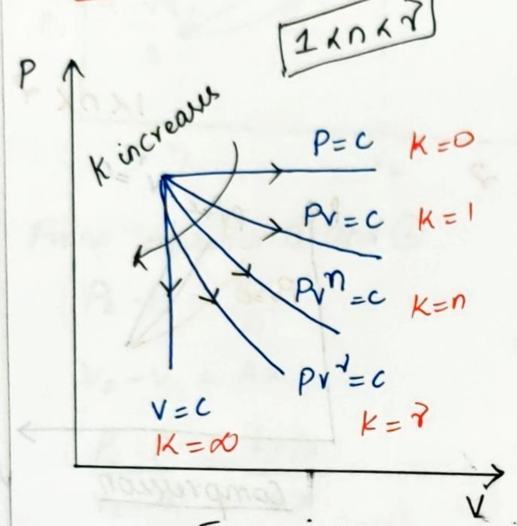
$\boxed{k = \gamma}$

5) polytropic process

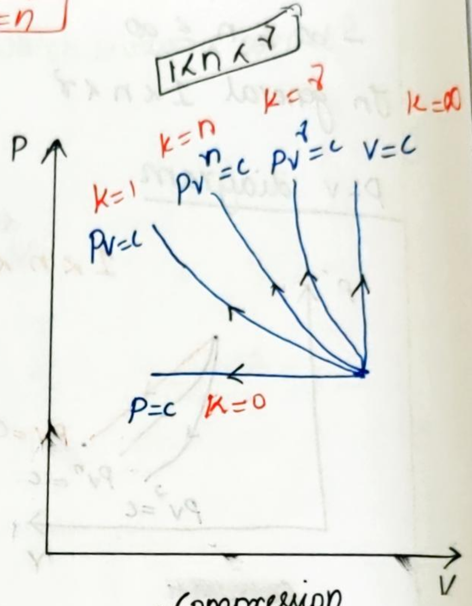
$PV^{n} = C$ (Any gas)

$PV^{k} = C$

$\boxed{k = n}$



Expansion



Compression

Energy of a System.

Energy of System

- Associated with complete system → Macroscopic energy
  - Kinetic Energy (KE)
  - Potential Energy (PE)
  - Electrical and magnetic Energy. — Negligible. 0
- Associated with individual molecules → Microscopic energy.
  - Translational Energy
  - Rotational Energy
  - Vibrational Energy
  - Bond Energy.

All these microscopic energy are combinely known as Internal energy

$$\boxed{E = \underbrace{K.E + P.E}_{\text{Macroscopic}} + \underbrace{U}_{\text{Microscopic}}}$$

To find the change in energy

$$E_1 = KE_1 + PE_1 + U_1$$

$$E_2 = KE_2 + PE_2 + U_2$$

$$E_2 - E_1 = (KE_2 - KE_1) + (PE_2 - PE_1) + (U_2 - U_1)$$

$$\boxed{\Delta E = \Delta KE + \Delta PE + \Delta U}$$

Conclusion

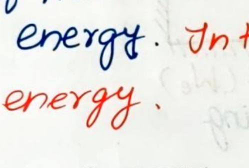
Internal energy depends on the size of the system. Larger the size of the system, higher will be the number of molecules hence higher will be the internal energy. Internal energy is an extensive energy.

STATIONARY SYSTEM

Stationary system means closed system

Block is stationary $\frac{1}{2}mv^2 = 0$

Closed system
- → Negligible change in KE $\boxed{\Delta KE = 0}$
- → Negligible change in PE $\boxed{\Delta PE = 0}$ $mgh = 0$



$1 \rightarrow 2$

$\Delta KE = 0$

$\Delta PE = 0$

$$\Delta E = \Delta \cancelto{0}{KE} + \Delta \cancelto{0}{PE} + \Delta U$$

$\boxed{\Delta E = \Delta U}$ (Stationary system)

* A stationary system is a closed system having no change in KE and PE during process.

* For stationary system change in energy is equal to the change in internal energy.